A versatile tree derivation procedure system for multivalent and paraconsistent inference.

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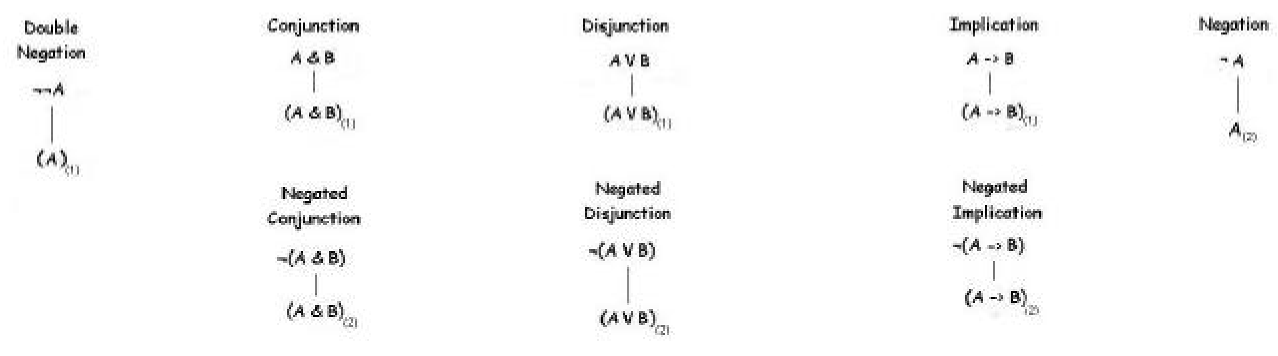
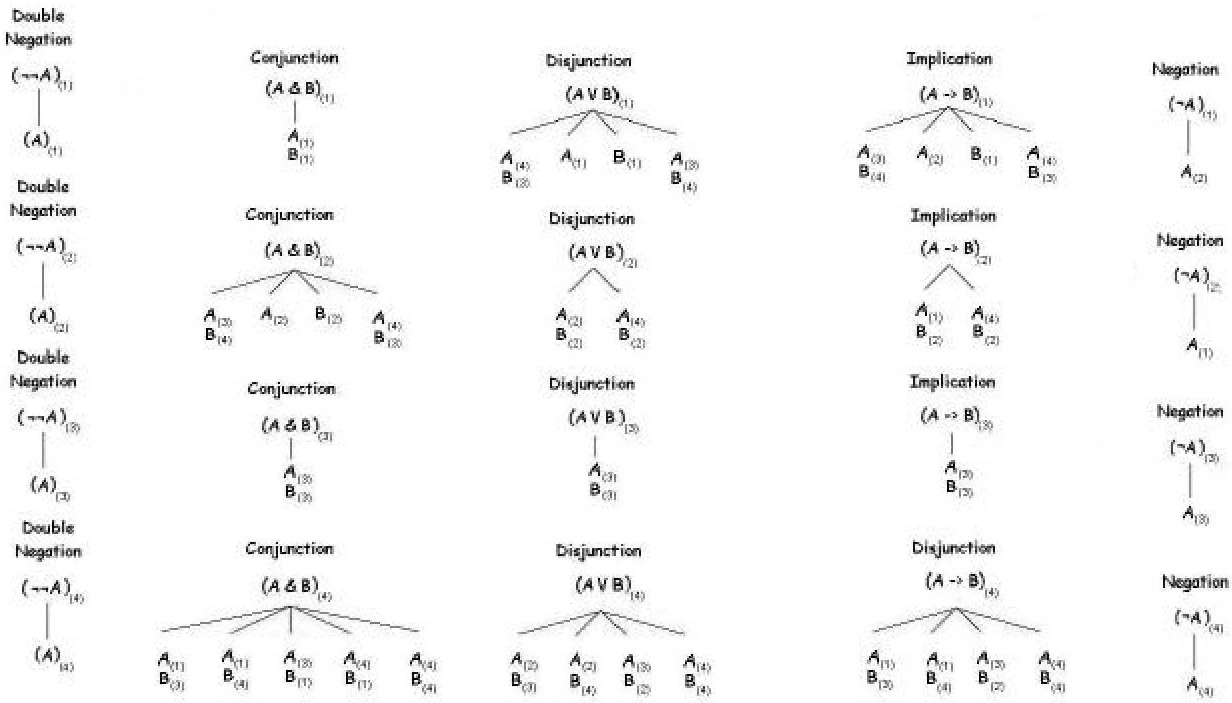
Abstract

The use of trees as a derivation procedure, which is a refinement of Beth's semantic tableaux method and Gentzen's sequent calculus, is an elegant and well established technique. Despite widespread acceptance and application, little effort has been made to extend the use of tree derivation procedures to multivalent alternatives to classical logic and none at all, so far as I am aware, to paraconsistent systems. The purpose of this paper is to outline briefly a single tree derivation procedure which was designed to work with a particular multivalued paraconsistent system: Epsilon $_{442}$[1]*, but will, with slight modification, work generally. The tree based derivation scheme presented provides a straightforward means by which truth-functional multivalued and paraconsistent reasoning systems may be automated. In addition, it offers a tractable way of producing proofs in a teaching situation.*

A summary of the decomposition rules for Epsilon 442 and a brief description of the process by which such rules can in general be produced is now given:

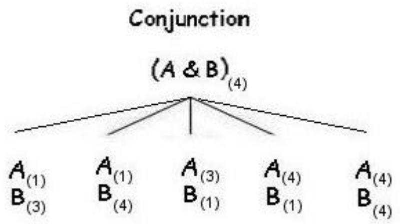


[1] See, Anderson, C.D.P., "Developing a framework for investigating inconsistency handling in automated reasoning.", 6th World Multi-Conference on Systemics, Cybernetics and Informatics (SCI 2002), Orlando, Florida , 2002.



How do we arrive at exactly these rules and how would we develop rules for a different system? Rather than painstakingly going through each structure in turn I will take a single example in order to demonstrate the principle. Take $(A \,\&\, B)_{(4)}$. To devise the appropriate decomposition structure we need to consult the Epsilon 442 matrix for conjunction:

(A & B)	1	2	3	4
1	1	2	4	4
2	2	2	2	2
3	4	2	3	2
4	4	2	2	4



We can readily see that (A & B) is assigned the value 4 in just five cases each of these is represented by its own branch. Each of the other structures is arrived at in a similar fashion. Of course, different matrices would give rise to a different decomposition rule but the process remains straightforward.

Full Matrices for Epsilon 442[2]

NOT	
1	2
2	1
3	3
4	4

AND	1	2	3	4
1	1	2	4	4
2	2	2	2	2
3	4	2	3	2
4	4	2	2	4

OR	1	2	3	4
1	1	1	1	1
2	1	2	4	4
3	1	4	3	1
4	1	2	1	4

COND	1	2	3	4
1	1	2	4	4
2	1	1	1	1
3	1	4	3	1
4	1	2	1	4

[2] NB. In Epsilon 442, {1,3} are designated values, {2} is anti-designated and {4} is non-designated