Proof Methods for Combinations of Logics

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Introduction

Temporal logic or dynamic logics have been used to capture change over time or the dynamic aspects of systems. Modal logics have been used to capture to capture notions such as the belief, knowledge, desires or intentions of a system. Together, combinations of these logics allow the specification of complex systems such as distributed or multi-agent systems [FW97, Hal87, MvdH95], the analysis of security protocols [Syv93, GMP92], knowledge games [vD00] etc.

Proof Methods for the Fusion of Logics

In the Logic and Computation Group we are interested in proof methods for these complex combinations of logics. Initially we developed resolution based proof methods for the fusion of linear-time temporal logics and single or multi-modal S5 [DFW98, DF00b]. This involves the translation to a normal form which reduces the nesting of subformulae and reduces the number of operators down to a core set. In doing this we obtain clauses relating to the modal part, clauses relating to the initial moment in time, clauses specifying eventualities (eg $\diamondsuit p$ stating that p must hold now or in the future), and clauses that specify constraints on the next moment in time. This allows us to separate clauses for the temporal and modal dimensions and perform resolution separately on each set of clauses, passing information between each of the sets in the form of disjunctions of literals.

We also defined a tableau for the fusion of linear-time temporal logics and single or multi-modal S5 or KD45 [WDF98]. This is essentially the combination of tableau methods for propositional linear time temporal logics [Wol85] and that for the modal logics S5 and KD45 [HM92].

We have also developed resolution based proof methods for the branching-time temporal logic CTL fused with belief (KD45) in [DFB02].

Allowing Interactions

In this context, interaction axioms allow us to consider how knowledge evolves over time. For example, for certain classes of systems, interaction occurs between knowledge and time so that, for example, statements such as *"if an agent knows that in the next moment p holds then, in the next moment, the agent knows p holds"* are valid. Systems with this property have been termed as *synchronous* with *no forgetting*, or *unbounded memory* or *perfect recall* [FHMV95, HV88, HV89]. For other systems we may want the converse of this. Systems with this property are known as *synchronous* with *no learning* [FHMV95, HV88, HV89]. For many of the interactions described in the literature [HV88, HV89] this increases the complexity of the logic (sometimes dramatically) even for the single agent case.

We have defined resolution based proof methods for the single agent cases of synchrony and perfect recall [DF00a], and synchrony and no learning [NDF01]. These use the translation to normal form described above and add clauses to account for the synchrony and perfect recall or no learning axioms. Recently we have developed a tableau based proof method for temporal logics of knowledge with synchrony and perfect recall or synchrony and no learning which explicitly applies the relevant axiom to formulae prior to constructing the tableau [DNF03].

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