Using SPASS for proving theorems within Mereotopology

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We report work in progress about using the automated theorem prover SPASS [12] for proving theorems within mereotopology. The mereotopological theory used is a spatio-temporal extension of the spatial representation language RCC-8 [11]. RCC-8 is a theory of space and time with a primitive dyadic relation of *connection*. The theory has been developed in a series of papers [11, 2, 6]. The most distinctive feature of the theory is that it uses extended regions instead of points as fundamental entities. For the formal theory to be used for reasoning about changes in spatial relations, it was augmented with a set of *envisioning axioms*. These axioms specify which direct transitions can be made in the topological relations between pairs of regions. The change of state in RCC relations via potential motion, analyzed through transition graphs in which relations form a *conceptual neighbourhood* was thus posited in [3] (see figure 2). Spatio-temporal continuity remained an implicitly assumed notion.

Galton [5] and thereafter Muller [10] looked at what continuity implies for a commonsense theory of motion. Muller had an explicit generic characterization of continuity and presented theorems for the (non-existence of) transitions between states in RCC-8 relations [10] within a space-time framework. But Muller's interpretation of theorems of (non-existence of) transitions has been shown not to be fully adequate [4]. Davis presents an alternative framework for characterisation of transitions. However it is not expressed as an object level first-order theory, and sacrifices the spirit of mereotopology as it defines time instants and spatio-temporal points.

We present a more comprehensive framework. The basic entities of our theory are non empty regions of space-time. Following [7], space-time regions are termed *histories*. Note that following previous work within the group [11, 6] we do not wish to admit lower dimensional entities - e.g. in our work on spatial mereotopology all regions were of the same dimension and we did not consider boundaries as spatial entities. Here too we do not admit lower dimensional entities such as temporal points into our ontology for the same reasons as argued in [11, 2]. In order to refer to regions within a given time we introduce the notion of *temporal slice*. Thus $\mathsf{TS}(x, y)$ says that x is a temporal slice of y and the syntactic sugar $\frac{y}{t}$ denotes the temporal slice of y during t. Fig. 1 shows a space-time history for a 2-D object and a temporal slice of the history. In a n-D space, a space-time history is a n+1 dimensional volume.

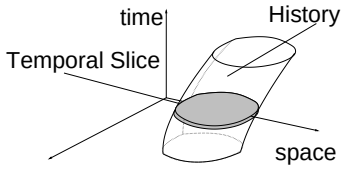

Figure 1: A s-t history is a n+1 dimensional volume in a n-D space.

Our theory is based on the basic dyadic relation of connection. We have three versions of connection relation: spatial, temporal and spatio-temporal interpreted in pure space, time and space-time respectively (see [8] for details). We introduce operators to characterize transitions. Furthere we have characterized a number of distinct notions of space-time continuity [8] including the notion of *strong firm-continuity* on which figure 2 depends.

The extended spatio-temporal theory is a first-order theory with equality. To formalize our theory we use a sorted first-order logic based on the logic LLAMA [1]. The principal sorts we use are REGION and NULL. We define a set of Boolean functions. The Boolean functions are sum, difference, intersection and complement. The functions are partial but as discussed in [2], are made total in the sorted logic by specifying sortal restrictions and letting the result sort of the partial functions be REGION $\cup$ NULL. The functions can be regarded as genuine Boolean operators

over the domain REGION $\cup$ NULL. In the SPASS input files along with the definitions for the Boolean functions (mentioned here), we add axioms for an equational theory of Boolean algebra [adapted from [9]] and have axioms to link the Boolean algebra to the relational part of the theory. This allows axioms for the Boolean functions to be added without loss of saturation. Although previously [8] we defined $\frac{y}{t}$ as syntactic sugar, for computational reasons, in SPASS $\frac{y}{t}$ is represented using an actual binary function, with axioms to specify its semantics.

In the absence of intended models and a syntactic proof of completeness, the only way to know that the axiomatisation fulfills our intentions to characterize continuous transitions is by proving 'transition theorems'. There are two aspects to proving the correctness of the conceptual neighbourhood: the links that are present need to be shown to be necessary, and those absent to be shown to represent discontinuous transitions. The latter is a theorem proving task but the former requires model building and appears to be much harder automated. We confine ourselves to the second task here.

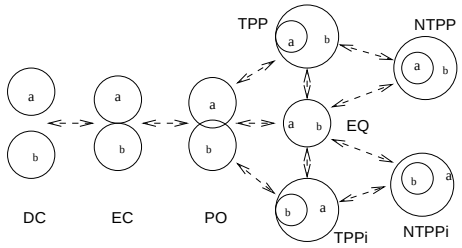

Figure 2: Conceptual Neighbourhood for RCC-8 relations under strong firm continuity.

The *non-existence theorems* formally show which transitions are not continuous and correspond to the links absent in the transition graph shown in figure 2 for RCC-8 under strong frim continuity. We are using the automated theorem prover SPASS to prove the non-existence transition theorems. The clausal form of the underlying theory consists of 571 clauses, 80 predicates and 219 function symbols. http://www.comp.leeds.ac.uk/qsr/SpassProofs list some proofs found by SPASS in our spatio-temporal theory.

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References

[1] A G Cohn, 'A more expressive formulation of many sorted logic', *Journal of Automated Reasoning*, **3**, 113–200, (1987).

[2] A G Cohn, B Bennett, J Gooday, and N Gotts, 'RCC: A Calculus for Region based Qualitative Spatial Reasoning', *GeoInformatica*, **1**(3), 275–316, (1997).

[3] A G Cohn, N M Gotts, Z Cui, D A Randell, B Bennett, and J M Gooday, 'Exploiting temporal continuity in qualitative spatial calculi', in *Spatial and Temporal Reasoning in Geographical Information Systems*, ed., M J Egenhofer et al., pp. 5–24. Oxford University Press, (1998).

[4] E Davis, 'Continuous shape transformation and metrics of shape', *Fundamenta Informaticae*, **46**(1-2), 31–54, (2001).

[5] A P Galton, *Qualitative Spatial Change*, Oxford University Press, 2000.

[6] N M Gotts, J M Gooday, and A G Cohn, 'A connection based approach to common-sense topological description and reasoning', *The Monist*, **79**(1), 51–75, (1996).

[7] P J Hayes, 'The Second Naive Physics Manifesto', in *Formal Theories of the Commonsense World*, ed., J. R. Hobbs et al., 1–36, Ablex, (1985).

[8] S M Hazarika and A G Cohn, 'Qualitative spatio-temporal continuity', in *Proc. of COSIT'01*, number 2205 in LNCS, pp. 92–107, (2001).

[9] K. Kuratowski, *Introduction to Set Theory and Topology*, Pergamon Press, 2nd edn., 1972.

[10] P. Muller, 'A qualitative theory of motion based on spatio-temporal primitives', in *Proc. of KR'98*, ed., A. G. Cohn et al., pp. 131–141, (1998).

[11] D. A. Randell, Z. Cui, and A. G. Cohn, 'A spatial logic based on regions and connection', in *Proc. of KR'92*, pp. 165–176, (1992).

[12] C. Weidanbach, 'SPASS: Combining superposition, sorts and splitting', in *Handbook of Automated Reasoning*, eds., A Robinson and A Voronkov, Elsevier Sciene Publishers. SPASS is available from `http://spass.mpi-sb.mpg.de/`.