Temporalising tableaux

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First-order temporal logic (FOTL) based on the flow of time $\langle \mathbb{N}, < \rangle$ is notorious for its 'bad computational behaviour:' even the two-variable monadic fragment of this logic is not recursively enumerable (see e.g. [2] and references therein). A certain breakthrough has been recently achieved in [2], where the so-called *monodic fragment* of FOTL is introduced by restricting applications of temporal operators to formulas with at most one free variable. The full monodic fragment (containing full first-order logic) turns out to be axiomatisable [3]. Moreover, by restricting its first-order part to decidable fragments, we obtain decidable monodic FOTLs, say, the monodic guarded, monodic two-variable, and monodic monadic fragments. This opens a way to various applications of the monodic FOTL in knowledge representation, temporal databases, and other fields. For example, many temporal description logics and spatio-temporal logics can be regarded as fragments of monodic FOTL [4, 5, 6]. Unfortunately, the decision procedures provided in [2] are of model-theoretic character and cannot be used as a basis for implementations. In [1] a resolution-based approach was developed for fragments weaker than monodic fragments. A *tableau-based analysis* of the decision problem for monodic FOTL has been missing. In this paper we are trying to fill in this gap. More specifically, our aims are as follows:

1. *to develop a general framework for devising tableau-based decision procedures for decidable monodic FOTLs and then,*
2. *within this framework, to construct tableau systems for a number of concrete monodic fragments.*

We consider monodic FOTLs interpreted in models with both expanding and constant domains. The former case is technically much easier, but the latter one is more general: reasoning with expanding domains can be reduced to reasoning with constant domains.

Our approach is based on the following ideas:

- *modularity*—a decision procedure for a given fragment of first-order logic is combined with Wolper's tableaux [7] for propositional temporal logic (PTL);
- *finite quasimodel* representations of temporal models with potentially infinite first-order domains;
- the *minimal type* technique to keep the domains of temporal models constant.

To achieve modularity, we separate the temporal and the pure first-order parts of formulas and treat them using available procedures for PTL and the corresponding first-order fragment. This is done

by replacing all occurrences of formulas starting with temporal operators by their 'surrogates'—unary predicates (the language is monodic) the proper 'temporal behaviour' of which is ensured by some auxiliary *surrogate axioms*. The resulting set of purely first-order formulas is treated then by the given first-order decision procedure to obtain descriptions of all possible models for this set. If the procedure fails, then the formula is not satisfiable. Otherwise we make one 'temporal step,' namely, omit the 'next-time' operator (as in Wolper's tableaux) and add new surrogate axioms. This yields another set of purely first-order formulas, an so forth.

Some special techniques are used to preserve the representation of tableaux finite and to guarantee termination. For example, first-order models are represented by finite sets of *types*, each of which standing for possibly infinite number of domain elements. *Quasimodels* are used to encode temporal models by associating a finite set of types with each time instant.

When a tableau is completed, the pruning technique (again, as in Wolper's tableaux) is used to ensure that all eventualities are fulfilled. Then the resulting tableau represents quasimodels of the testing formula.

A rather general theorem provides conditions under which a first-order decision procedure can be combined with Wolper's tableaux to yield a tableau-based decision procedure for the corresponding monodic FOTL. The price we have to pay for this level of generality is that the resulting combined tableaux are far from optimal. In particular, in many concrete cases new rules can be used instead of surrogate axioms. Thus, the general framework for combining tableaux is not supposed for direct applications or implementations, but rather as a *guide* for considering more specific cases.

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