The decidability of the first-order theory of the Knuth-Bendix orders in the case of unary signatures

Konstantin Korovin and Andrei Voronkov

University of Manchester
{korovin,voronkov}@cs.man.ac.uk

Introduction of ordering constraints has been one of the main breakthroughs in the saturation based theorem proving. Using solvability of ordering constraints we can dramatically reduce the number of redundant inferences in a resolution-based prover. As a consequence, the problem of solving ordering constraints for the known simplification orders is one of the important problems in the area. A simplification order is a total monotonic order on ground terms. Given such an order we can consider ordering constraints which are quantifier-free formulas in the language of the term algebra with equality and order. Two kinds of orders are used in automated deduction: the Knuth-Bendix orders [5] and various versions of the recursive path orders [3]. Because of its importance, the decision problem for ordering constraints has been well-studied. For the recursive path orders decidability and complexity issues were considered in [4, 1, 11, 12, 10, 9]. For the Knuth-Bendix orders we have the following results: the decidability of constraints [6], a nondeterministic polynomial-time algorithm for constraint solving [7], a polynomial-time algorithm for solving constraints consisting of a single inequality [8].

In resolution-based theorem proving there are important simplifications which allow us to remove clauses form the search space (for example subsumption). It turns out that in order to express applicability conditions for these simplifications, we need to consider constraints which involve first-order quantifiers. Unfortunately the first-order theory of the recursive path orders is undecidable [13, 2]. Only recently the decidability of the first-order theory of recursive path orders in the case of unary signatures has been proven [9]. A signature is called unary if it consists of unary function symbols and constants.

Continuing our work on the Knuth-Bendix orders we prove the decidability of the first-order theory of the Knuth-Bendix orders in the case of unary signatures.

Theorem 1. *The first order theory of any Knuth-Bendix order for a unary signature is decidable.*

Our decision procedure uses interpretation of unary terms as trees and uses decidability of the weak monadic second-order theory of binary trees.

Let us note that the decidability problem for the first-order theory of the Knuth-Bendix orders in the the case of arbitrary signatures remains open.

References

1. H. Comon. Solving symbolic ordering constraints. *International Journal of Foundations of Computer Science*, 1(4):387–411, 1990.
2. H. Comon and R. Treinen. The first-order theory of lexicographic path orderings is undecidable. *Theoretical Computer Science*, 176(1-2):67–87, 1997.
3. N. Dershowitz. Orderings for term rewriting systems. *Theoretical Computer Science*, 17:279–301, 1982.
4. J.-P. Jouannaud and M. Okada. Satisfiability of systems of ordinal notations with the subterm property is decidable. *Lecture Notes in Computer Science*, 510:455–468, 1991.
5. D. Knuth and P. Bendix. Simple word problems in universal algebras. In J. Leech, editor, *Computational Problems in Abstract Algebra*, pages 263–297. Pergamon Press, Oxford, 1970.
6. K. Korovin and A. Voronkov. A decision procedure for the existential theory of term algebras with the Knuth-Bendix ordering. In *Proc. 15th Annual IEEE Symp. on Logic in Computer Science*, pages 291–302, Santa Barbara, California, June 2000.
7. K. Korovin and A. Voronkov. Knuth-bendix ordering constraint solving is NP-complete. In F. Orejas, P.G. Spirakis, and J. van Leeuwen, editors, *Automata, Languages and Programming, 28th International Colloquium, ICALP 2001*, volume 2076 of *Lecture Notes in Computer Science*, pages 979–992. Springer Verlag, 2001.
8. K. Korovin and A. Voronkov. Verifying orientability of rewrite rules using the Knuth-Bendix order. In A. Middeldorp, editor, *Rewriting Techniques and Applications, 12th International Conference, RTA 2001*, volume 2051 of *Lecture Notes in Computer Science*, pages 137–153. Springer Verlag, 2001.
9. Narendran and Rusinowitch. The theory of total unary rpo is decidable. In *First International Conference on Computational Logic*, 2000.
10. Narendran, Rusinowitch, and Verma. RPO constraint solving is in NP. In *CSL: 12th Workshop on Computer Science Logic*. LNCS, Springer-Verlag, 1998.
11. R. Nieuwenhuis. Simple LPO constraint solving methods. *Information Processing Letters*, 47:65–69, 1993.
12. R. Nieuwenhuis and J.M. Rivero. Solved forms for path ordering constraints. In *In Proc. 10th International Conference on Rewriting Techniques and Applications (RTA)*, volume 1631 of *Lecture Notes in Computer Science*, pages 1–15, Trento, Italy, 1999.
13. Treinen. A new method for undecidability proofs of first order theories. *FSTTCS: Foundations of Software Technology and Theoretical Computer Science*, 10, 1990.