Resolution for Temporal Logics of Knowledge with Interactions

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Combined temporal and modal logics have been used in computer science for reasoning about complex situations such as, for instance, the specification and verification of distributed [5] and multi-agent systems [9]. We consider a particular combination, the fusion of PTL and $\mathsf{S5}_{(n)}$, a *Temporal Logic of Knowledge* ($\mathsf{KL}_{(n)}$) in which the dynamic and informational components of such systems can be represented. Given a logical characterisation of a system, we then want to *prove* that it matches the desired requirements and properties. Although there are proof methods for such logics [5, 4, 3, 7], most of these cannot be easily adapted to deal with interactions between the temporal and epistemic dimensions.

We say that time and knowledge interact when axioms relating the temporal and epistemic dimensions are added to the logic. For instance, the axioms $\mathrm{K}_i \bigcirc\varphi \Rightarrow \bigcirc\mathrm{K}_i \varphi$ (*perfect recall and synchrony*) and $\bigcirc\mathrm{K}_i \varphi \Rightarrow \mathrm{K}_i \bigcirc\varphi$ (*no learning and synchrony*) [5] express how the knowledge of an agent evolves over time: the first models situations at which agents remember all their past history, whilst the second models situations at which agents do not distinguish their futures. Adding interacting axioms usually increases the complexity of the validity problem for the combined logic. For instance, on the single-agent case, the validity problem for the combined logic without interactions, $\mathsf{KL}_{(1)}$, is PSPACE, whereas if, in particular, the *no learning and synchrony* interaction axiom is added, the complexity becomes EXPSPACE [6].

Recently, it has been shown that the resolution method for $\mathsf{KL}_{(n)}$ proposed in [3] can also be applied, with few modifications, to deal with perfect recall and synchrony [2] and with no learning and synchrony [8]. Instead of adding new resolution rules, which would make implementation more difficult, we introduce a set of clauses, which make explicit the constraints imposed by the interaction axioms. Also, a number of definitions have to be added to the set of clauses before the resolution method is performed. Many of these definitions are used for renaming complex formula in the scope of the knowledge operator (K_i).

We have now proposed a new normal form, which avoids the introduction of these definitions for complex formulae, and so reducing the set of clauses to which the resolution method is applied. This new normal form was inspired by that proposed in [1], which simplifies the resolution method for temporal logics. We then benefit from this improvement in the method for dealing with the temporal dimension, and have adapted the resolution rules for the knowledge dimension. The resulting proof method for $\mathsf{KL}_{(n)}$ is terminating, sound and complete. Proving correctness of the extended method for synchronous systems with perfect recall or no learning is ongoing work.

We are also investigating how to extend the basic method to deal with interactions of the form $\bigcirc^l\mathrm{K}_i \phi \Rightarrow \mathrm{K}_i \bigcirc^r\phi$ and $\mathrm{K}_i \bigcirc^l\phi \Rightarrow \bigcirc^r\mathrm{K}_i \phi$, where $l, r \in \mathbf{N}$, $\bigcirc^0\phi = \phi$, and $\bigcirc^i\phi = \bigcirc\bigcirc^{i-1}\phi$ for $i > 0$. Whilst, in general, interactions of these forms have not been studied in computer science, some of their instances, such as the above-mentioned axioms, are very well known, as they express important properties of multi-agent systems. This is the first step towards a general method for dealing with general interactions between modalities representing different dimensions of reasoning.

References

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