Logical Representation for Route Graphs

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Complex architectures involving combination of sensor input, automation and human control tend to make of robot navigation a field in which high-level spatial representation can play a significant role.

In [WKB00, RL02] *route graphs* have been proposed as a hybrid topological-metric data-structure for robot navigation. Graph nodes represent the *places*, either regions or landmarks, that may be associated to decision points in the navigation. Graph edges represent segments of the *routes* that a robot may follow. Places and routes may be described at different levels of granularity. For example, a region — like a room — might be associated to a set of landmarks — such as a door, a window, a desk — with route segments in between. Similarly, a building might be split into rooms, etc.. The actions of the robot may be described either in a qualitative way — pass through the door and turn left — or in numerical one — turn on the spot 90 degrees clockwise, go straight ahead for 2 meters.

In order to specify a spatial ontology that may be associated to a route graph, it can be useful to consider the underlying topology. A possibility which is compatible with the discreteness and granularity of graph representation, allowing for infinite models, is that of restricting to atomic, T_0 order topologies. These are just those topogies that are equivalent to partial orders as well as ultimately based on atoms (i.e. atomic open sets).

Some propositional super-intuitionistic logics can be used for region-based reasoning in such spaces — relying on an interpretation that associates formulæ to topological sets, along the line of [Tar56]. Those were in fact investigated in [TSB02, Tor04]. In particular, it was shown there how to represent connectivity relations (similar to the $RCC8$ relations in [Ben96]), properties that include connectedness as well as some of its stronger variants (strong connectedness, primeness), and some atom sets that can be defined through them — such as, given a region, the atoms connected to it.

Some of these notions can be quite significant in relation with route graphs. For example, landmarks may be associated to atoms quite naturally. Enclosed regions counting for local enviroments may be associated to prime sets. A route may be represented in terms of connection, and a dedicated access in terms of strong connection.

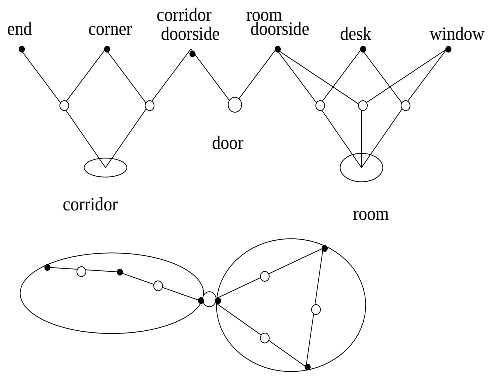

Figure 1:

The logical representations given in [Tor04] relied on the extension of intuitionistic propositional logic by means of an *S5*-style modality and a quantification ranging over formulæ. The latter made those encodings rather hard to deal with in practice, since second-order intuitionistic propositional logic is undecidable [Gab81].

However, it is possible to get around quantification by introducing specific spatial operators. The following are examples of how some of them can be expressed axiomatically, on top of an intuitionistic language $\rightarrow, \vee, \wedge, \perp$ — granted as usual $\sim$, $\leftrightarrow$ — extended with a strong *S5*-style modality, axiomatised in [TSB02, Tor04], where $\Diamond\,\alpha = \sim\Box\sim\alpha$ is used to express the *non-emptiness* of α (more precisely, of the associated open set). Alternatively, and more expressively, a combination of *S4* and strong *S5* could be used, as presented in [Ben96, AvB02].

1. The collection of *all atoms* (in the model) can be "flattened" and represented as a constant that satisfies the following:

   $\vdash \mathsf{atoms} \rightarrow \alpha \vee \sim\alpha$

   $\vdash \sim\sim\mathsf{atoms}$

2. For the *connectedness* of α:

   $\vdash \mathsf{con}(\alpha) \rightarrow (\Box(\alpha \rightarrow \beta \vee \sim\beta) \rightarrow \Box(\alpha \rightarrow \beta) \vee \Box(\alpha \rightarrow \sim\beta))$

   $$\frac{\vdash \Box(\alpha \rightarrow \beta \vee \sim\beta) \rightarrow \Box(\alpha \rightarrow \beta) \vee \Box(\alpha \rightarrow \sim\beta)}{\vdash \mathsf{con}(\alpha)}$$

3. For the *strong connectedness* of α (for Δ set of propositions):

   $\vdash \mathsf{con}_s(\alpha) \rightarrow ((\alpha \rightarrow \beta \vee \sim\beta) \rightarrow (\alpha \rightarrow \beta) \vee (\alpha \rightarrow \sim\beta))$

   $$\frac{\Delta \vdash (\alpha \rightarrow \beta \vee \sim\beta) \rightarrow (\alpha \rightarrow \beta) \vee (\alpha \rightarrow \sim\beta)}{\Delta \vdash \mathsf{con}_s(\alpha)}$$

4. For the *primeness* of α:

   $\vdash \mathsf{prime}(\alpha) \rightarrow (\Box(\alpha \rightarrow \beta \vee \gamma) \rightarrow \Box(\alpha \rightarrow \beta) \vee \Box(\alpha \rightarrow \gamma))$

   $$\frac{\vdash \Box(\alpha \rightarrow \beta \vee \gamma) \rightarrow \Box(\alpha \rightarrow \beta) \vee \Box(\alpha \rightarrow \gamma)}{\vdash \mathsf{prime}(\alpha)}$$

5. In order to express that α is an *atom*:

   $\vdash \mathsf{atom}(\alpha) \leftrightarrow (\Diamond\,\alpha \;\wedge\; \mathsf{prime}(\alpha) \;\wedge\; (\alpha \rightarrow \mathsf{atoms}))$

For the purpose of qualitatively representing route graphs, it would be further necessary to extend the language in order to allow for a description of the dynamic aspect. Therefore, the topological representation based on the "vertical" order induced by inclusion, allowing for the static account of connectivity and granularity I have been referring to up to now, should be integrated by means of some "horizontal" relations between topological sets — such as the orthogonal pairs *east-west*, *north-south* and *left-right*, *in front-behind* — as well as by a notion of discrete branching time, in order to represent the orientation and the possible movements of the robot through time.

References

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