Tableaux for modal and temporal description logics

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Description logics are logical formalisms intended for representing knowledge about static application domains in a structured way (see e.g. [6] and references therein). In the basic description logic $\mathcal{ALC}$ [23], for example, we can use the concept

$$\mathsf{Person} \sqcap \exists \mathsf{lives\_in}.\mathsf{Bavaria} \sqcap \forall \mathsf{drinks}.\mathsf{Beer}$$

to describe the persons living in Bavaria and drinking only beer. (Here Person, $\mathsf{Bavaria}$, and Beer are concept names (unary predicates), and $\mathsf{lives\_in}$ and drinks are role names (binary predicates).) The knowledge (or rather belief) of an agent, say Tom, that all people living in Bavaria drink only beer and, conversely, that all people drinking only beer live in Bavaria can be represented then by the terminological axiom

$$\mathsf{Person} \sqcap \exists \mathsf{lives\_in}.\mathsf{Bavaria} = \mathsf{Person} \sqcap \forall \mathsf{drinks}.\mathsf{Beer}. \tag{1}$$

Modal logics are formal systems intended for reasoning with necessity-like and possibility-like operators, in particular, for reasoning about agents' knowledge of their knowledge bases and about the evolution of these knowledge bases in time (see e.g. [7] and references therein). Using the modal operators $[Tom\ believes]$ and $[always]$, we can relativize axiom (1) to represent the knowledge of agent Tom and also add to it some temporal flavor:

$$[Tom\ believes]\,(\mathsf{Person} \sqcap \exists \mathsf{lives\_in}.\mathsf{Bavaria} = \mathsf{Person} \sqcap [always]\forall \mathsf{drinks}.\mathsf{Beer}) \tag{2}$$

i.e., Tom believes that those people who live in Bavaria and nobody else drink only beer at any given moment of time. As just demonstrated, *modalized description logics* provide means for representing distributed knowledge about dynamic application domains. Such logics were constructed e.g. in [5, 22, 3, 4, 26, 28, 29, 30].

*The results presented in this paper are joint work with Carsten Lutz, Holger Sturm, and Frank Wolter.

Both description logics and modal logics are featured as 'a compromise between expressiveness and effectiveness', which led to the implementation of many of them as *representation and reasoning systems* [10, 14, 20]. Description logics with modal operators are not so well-behaved. The interaction between description logic constructs and modalities can substantially increase the complexity of reasoning tasks and even make them undecidable [4, 28, 19]. The series of papers [26, 28, 29, 30, 27] identified the following syntactical and semantical 'limits' within which interactions between the modal and the description parts of the combined logics usually preserve decidability:

- Modal operators can be applied to both concepts and terminological axioms (as in the example above) but not to roles.
- In the worlds of a Kripke model interpreting the modal operators, each object and concept name of the description logic can be interpreted both globally (i.e., by the same object or, respectively, set of objects in all worlds) and locally (i.e., by arbitrary objects or sets of objects).[1] Role names can be interpreted only locally.
- The domains of the worlds (on which we interpret the description logic component) can be assumed to be *constant*; the expressive power of the combined language makes it possible to simulate the *varying* or *expanding domain assumptions* which presuppose less interaction.

By allowing modalized or/and global roles, one can easily construct undecidable description logics with epistemic and temporal operators [27].

The proofs of decidability in [26, 29, 30] are based on a semantical approach. They do not give any 'practical' decision procedures having a potential to be implemented in reasoning systems which are reasonably fast on reasonably large sets of problems.

The aim of this paper is to construct a tableau decision algorithm for the modal description logic $\mathbf{K}_{\mathcal{ALC}}$ with constant domains. More precisely, we present a tableau procedure which is capable of deciding, given a formula φ of the form (2) (in which modal operators are applied only to concepts and TBox axioms, but not to roles), whether φ is satisfiable in a model with constant domains. Tableau-based algorithms have been shown to be 'practical' even for logics of rather high complexity [9, 13, 15]. This gives us grounds to believe that, although the satisfiability problem for $\mathbf{K}_{\mathcal{ALC}}$ is known to be NEXPTIME-complete [19], by providing a tableau decision algorithm we demonstrate that highly expressive description logics with modal operators have a chance to be implementable.

The logic $\mathbf{K}_{\mathcal{ALC}}$ *with expanding domains* was first considered by Baader and Laux [3] who proved its decidability by designing a tableau satisfiability-checking algorithm. However, they failed to construct such an algorithm working under

[1] If we represent a person, say Tom, by the object name tom then we most likely assume that this name is global, while the object name president_of_the_USA, representing the current president of the United States, should clearly be local.

the *constant domain assumption*, having left this as an open problem and explained some principle difficulties. Somewhat simplified, the idea of constructing tableaux for $\mathbf{K}_{\mathcal{ALC}}$ with expanding domains is as follows. Given a formula φ, we first apply to it the rules of a tableau system for $\mathcal{ALC}$, thus constructing an $\mathcal{ALC}$-model for the non-modal part of φ in the initial world w_0. Then we apply the rules of a tableau system for $\mathbf{K}$ to the modalized concepts and formulas in this model and construct a number of new worlds w_i 'populated' by *the same objects* as in w_0. After that we use the $\mathcal{ALC}$-rules in the worlds w_i and possibly *extend* their domains. And so forth. This straightforward approach does not work in the case of models with constant domains. Having expanded the domain of w_i with a new object a, we have to add a to the domain of w_0 which, in turn, may force us to expand this domain, and so the domains of the w_i as well, with some new objects, and so on (for more details consult [3]).

The main technical contribution of the paper is that it shows how to design a machinery for constructing tableaux with constant domains. The fundamental difference from the approach of [3] is that our tableau algorithm constructs not a model itself but its representation in the form of a *quasimodel*, the worlds in which are 'populated' by *types* of objects rather than real objects. Quasimodels were first introduced in [26] and used in [24] for designing a tableau procedure for temporal description logics with expanding domains. In fact, the transference of the notion of quasimodel from model theory in [26] to proof theory in [24] and the present paper provides a nice example of how an 'abstract' decidability proof can serve as an important basis for an implementable decidability proof.

We have chosen the basic modal description logic $\mathbf{K}_{\mathcal{ALC}}$ for treating in this paper only in order to make the ideas of the tableaux and the employed techniques as clear as possible. The developed methods can be extended to more sophisticated logics, say, **S4** or temporal logics based on $\mathcal{ALC}$ [18]. Moreover, the approach developed in this paper can be generalized to the monodic fragments of *first-order* modal and temporal logics [11, 31] by combining tableau procedures for their first-order and modal components in a *modular* way; for details visit `http://www.dcs.kcl.ac.uk/staff/mz`.

It is worth also noting that $\mathbf{K}_{\mathcal{ALC}}$ is closely related to the *Cartesian product* $\mathbf{K} \times \mathbf{S5}$ (cf. [8, 25]). Thus we obtain for free a tableau-based decision procedure for $\mathbf{K} \times \mathbf{S5}$ as well.

References

[1] A. Artale and E. Franconi. A temporal description logic for reasoning about actions and plans. *Journal of Artificial Intelligence Research (JAIR)*, (9), 1998.

[2] A. Artale and C. Lutz. A correspondance between temporal description logics. In P. Lambrix, A. Borgida, M. Lenzerini, R. Möller, and P. Patel-Schneider, editors, *Proceedings of the International Workshop on Descrip-*

tion Logics (DL'99), number 22 in CEUR-WS, pages 145–149, Linkoeping, Sweden, July 30 – August 1 1999.

[3] F. Baader and A. Laux. Terminological logics with modal operators. In C. Mellish, editor, *Proceedings of the 14th International Joint Conference on Artificial Intelligence*, pages 808–814, Montréal, Canada, 1995. Morgan Kaufmann.

[4] F. Baader and H.-J. Ohlbach. A multi-dimensional terminological knowledge representation language. *J. Applied Non-Classical Logics*, 5:153–197, 1995.

[5] F. M. Donini, M. Lenzerini, D. Nardi, and A. Schaerf. Adding epistemic operators to concept languages. In W. Nebel, Bernhard; Rich, Charles; Swartout, editor, *Proceedings of the 3rd International Conference on Principles of Knowledge Representation and Reasoning*, pages 342–356, Cambridge, MA, Oct. 1992. Morgan Kaufmann.

[6] F. M. Donini, M. Lenzerini, D. Nardi, and A. Schaerf. Reasoning in description logics. In G. Brewka, editor, *Foundation of Knowledge Representation*, pages 191–236. CSLI-Publications, 1996.

[7] R. Fagin, J. Halpern, Y. Moses, and M. Vardi. *Reasoning about Knowledge.* MIT Press, 1995.

[8] D. Gabbay and V. Shehtman. Products of modal logics, part 1. *Journal of the IGPL*, 6:73–146, 1998.

[9] V. Haarslev and R. Möller. An empirical evaluation of optimization strategies for ABox reasoning in expressive description logics. In P. Lambrix, A. Borgida, M. Lenzerini, R. Möller, and P. Patel-Schneider, editors, *Proceedings of the International Workshop on Description Logics (DL'99)*, number 22 in CEUR-WS, pages 115–119, Linkoeping, Sweden, July 30 – August 1 1999.

[10] V. Haarslev and R. Möller. RACE system description. In P. Lambrix, A. Borgida, M. Lenzerini, R. lf Möller, and P. Patel-Schneider, editors, *Proceedings of the International Workshop on Description Logics*, number 22 in CEUR-WS, pages 140–141, Linkoeping, Sweden, July 30 – August 1 1999.

[11] I. Hodkinson, F. Wolter, and M. Zakharyaschev. Decidable fragments of first-order temporal logics. *Annals of Pure and Applied Logic*, 2000.

[12] I. Horrocks. *Optimising Tableaux Decision Procedures for Description Logics.* Phd thesis, University of Manchester, 1997.

[13] I. Horrocks. Using an expressive description logic: Fact or fiction? In A. Cohn, L. Schubert, and S.C.Shapiro, editors, *Principles of Knowledge Representation and Reasoning – Proc. of the Sixth International Conference*

KR'98, pages 636–647, Trento, Italy, June 2–5, 1998. Morgan Kaufmann Publ. Inc., San Francicso, CA, 1998.

[14] I. Horrocks. FaCT and iFaCT. In P. Lambrix, A. Borgida, M. Lenzerini, R. lf Möller, and P. Patel-Schneider, editors, *Proceedings of the International Workshop on Description Logics*, number 22 in CEUR-WS, pages 133–135, Linkoeping, Sweden, July 30 – August 1 1999.

[15] I. Horrocks, P. Patel-Schneider, and R. Sebastiani. An analysis of empirical testing for modal decision procedures. *Logic Journal of the IGPL*, 8(3):293–323, 2000.

[16] I. Horrocks, U. Sattler, and S. Tobies. Practical reasoning for expressive description logics. In H. Ganzinger, D. McAllester, and A. Voronkov, editors, *Proceedings of the 6th International Conference on Logic for Programming and Automated Reasoning (LPAR'99)*, number 1705 in Lecture Notes in Artificial Intelligence, pages 161–180. Springer-Verlag, Sept. 1999.

[17] U. Hustadt and R. A. Schmidt. Issues of decidability for description logics in the framework of resolution. In R. Caterra and G. Salzer, editors, *Automated Deduction in classical and non-classical logic*, volume 1761 of *Lecture Notes in Artificial Intelligence*, pages 191–205. Springer-Verlag, 1996.

[18] C. Lutz, H. Sturm, F. Wolter, and M. Zakharyaschev. A tableau calculus for temporal description logic: the constant domain case. Submitted. 2001.

[19] M. Mosurovic and M. Zakharyaschev. On the complexity of description logics with modal operators. In P. Kolaitos and G. Koletos, editors, *Proceedings of the 2nd Panhellenic Logic Symposion*, pages 166–171, Delphi, Greece, 1999.

[20] P. Patel-Schneider. DLP. In P. Lambrix, A. Borgida, M. Lenzerini, R. lf Möller, and P. Patel-Schneider, editors, *Proceedings of the International Workshop on Description Logics*, number 22 in CEUR-WS, pages 140–141, Linkoeping, Sweden, July 30 – August 1 1999.

[21] K. D. Schild. A correspondence theory for terminological logics: Preliminary report. In *Proc. of the 13^{th} IJCAI*, pages 466–471, Sidney, Australia, 1991.

[22] K. D. Schild. Combining terminological logics with tense logic. In M. Filgueiras and L. Damas, editors, *Progress in Artificial Intelligence – 6th Portuguese Conference on Artificial Intelligence, EPIA'93*, Lecture Notes in Artificial Intelligence, pages 105–120, Porto, Portugal, Oct. 1993. Springer-Verlag.

[23] M. Schmidt-Schauß and G. Smolka. Attributive concept descriptions with complements. *Artificial Intelligence*, 48(1):1–26, 1991.

[24] H. Sturm and F. Wolter. A tableau calculus for temporal description logic: The expanding domain case. Journal of Logic and Computation, 2001.

[25] F. Wolter. The product of converse PDL and polymodal K. *Journal of Logic anf Computation*, 10:223–251, 2000.

[26] F. Wolter and M. Zakharyaschev. Satisfiability problem in description logics with modal operators. In A. G. Cohn, L. Schubert, and S. C. Shapiro, editors, *KR'98: Principles of Knowledge Representation and Reasoning*, pages 512–523. Morgan Kaufmann, San Francisco, California, 1998.

[27] F. Wolter and M. Zakharyaschev. Modal description logics: modalizing roles. *Fundamenta Informaticae*, 39:411–438, 1999.

[28] F. Wolter and M. Zakharyaschev. Multi-dimensional description logics. In D. Thomas, editor, *Proceedings of the 16th International Joint Conference on Artificial Intelligence (IJCAI-99-Vol1)*, pages 104–109, S.F., July 31–Aug. 6 1999. Morgan Kaufmann Publishers.

[29] F. Wolter and M. Zakharyaschev. Temporalizing description logic. In D. Gabbay and M. de Rijke, editors, *Frontiers of Combining Systems*, pages 379–402. Studies Press/Wiley, 1999.

[30] F. Wolter and M. Zakharyaschev. Dynamic description logic. In K. Segerberg, M. de Rijke, H. Wansing, and M. Zakharyaschev, editors, *Advances in Modal Logic, Volume 2*. CSLI Publications, 2000.

[31] F. Wolter and M. Zakharyaschev. Decidable fragments of first-order modal logics. *Journal of Symbolic Logic*, 2001.