Optimising minimal Herbrand model generation procedures for guarded formulae

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Abstract. In recent work [9, 10] we devised a hyperresolution-based decision procedure for subfragments of the guarded fragment and their extensions, and for the corresponding clausal classes. A significant computational property of the procedure is that it explicitly generates all Herbrand models of the formula under consideration. However, the procedure does not immediately generate all minimal Herbrand models. We have previously proposed three refinements of hyperresolution which ensure that all and only minimal Herbrand models are generated [8]. All refinements rely on additional model constraint propagation or local minimality test which lead to pruning all non-minimal Herbrand models. A method that employs branching lemmas and leads to either omitting or restricting the applications of model constraint propagation and the local minimality test is introduced in [11]. The aim of the current work is to study how this method relates to our framework.

Model generation is a task complementary to refutational theorem proving. Its importance is as a result of potential application areas such as disproving conjectures in classical theorem proving [7], natural language [2], deductive databases [6, 17], and model-based diagnosis [1]. Characterizing classes of inputs for which termination of the derivation is guaranteed, defining appropriate formalisms for representing such models, and providing algorithms for working with the resulting model representations is recognized as a challenge in automated deduction [5, 15].

The advantage of hyperresolution decision procedures for satisfiable causal sets of range restricted clauses [4], as well as for clausal classes resulting from formulae in classical and non-classical logic [3, 11–13, 16], for satisfiable syntactically defined clause classes [5, 14, 10], and for clausal classes originating from guarded formulae [9] is that when the derivation terminates without finding a refutation proof and outputs a counter model, the user is provided with information, suitable for subsequent optimisations.

In previous work we have shown that hyperresolution generates all Herbrand models for a subfragment of the guarded fragment $\mathrm{GF1}^-$ and extensions of $\mathrm{GF1}^-$ and that not all of these models are minimal [8–10]. We have proposed three ways of refining hyperresolution so that it is a minimal model generation procedure which constructs all and only minimal Herbrand models for $\mathrm{GF1}^-$. The minimal Herbrand model generation procedures, specified by these refinements, have concrete application domains and are also relevant for multi-modal and description logics that can be embedded into $\mathrm{GF1}^-$.

However, all previously considered minimal Herbrand model generation procedures rely on either model constraint propagation or local minimality tests in addition to (complement) splitting and involve propagation of negative clauses rendered by already generated Herbrand models. These negative clauses are added dynamically every time a Herbrand model is generated and if there is another open branch of the derivation tree the Herbrand model, represented by this branch is tested for minimality. The disadvantage is that previously generated Herbrand models which are subsequently propagated can be large. Moreover, in some cases Herbrand model generation procedures perform unnecessary minimality tests on Herbrand models that are already minimal. Therefore the aim of our current work is to restrict the range of the applicability of a local minimality test and in some cases even omit it. We follow the approach described in [11] and adopt it to our framework.

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